TDC Odd Semester Exam., 2020
held in July, 2021

COMMERCE

(Honours)

(3rd Semester)

Course No. : BCMH–301

(Business Mathematics)

Full Marks : 50
Pass Marks : 17

Time : 2 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, taking **one** from each Unit

UNIT—I

1. *(a)* In how many ways can the letters of the word 'DAUGHTER' be arranged so that the vowels are always separated? 3

(b) Find the term independent of x in the expansion of $\left(x+\frac{1}{x}\right)^{10}$. 3

(c) If

$$P=\begin{bmatrix}9 & 1\\ 4 & 3\end{bmatrix} \text{ and } Q=\begin{bmatrix}1 & 5\\ 7 & 12\end{bmatrix}$$

then find the matrix R such that $5P+3Q+2R$ is a null matrix. 4

2. *(a)* If $(x)^{\sqrt{x}}=(\sqrt{x})^{x}$, then show that $x=4$. 3

(b) Find the value of $\sqrt{e}$ correct up to four places of decimal. 3

(c) Define binomial theorem. Indicating the general term, write down the expansion of $(a+x)^{n}$, where n is a positive integer. 2+2=4

Or

Find the rank of the following matrix : 4

$$\begin{bmatrix}1 & 2 & 3\\ 3 & 4 & 5\\ 4 & 5 & 6\end{bmatrix}$$

UNIT—II

3. *(a)* Define the continuity of a function at $x = a$. A function $f(x)$ is defined as follows :

$$f(x) = \begin{cases} 1 - x, & 0 \le x \le 1 \\ x - 1, & x > 1 \end{cases}$$

Find $f\left(\frac{1}{2}\right)$ and examine the continuity of $f(x)$ at $x = 0$. 1+3=4

(b) Using first principle, find the derivative of $3x^3 + 7$. 3

(c) Show that the function

$$f(x) = x^3 - 3x^2 + 3x + 1$$

is neither a maximum nor a minimum at $x = 1$. 3

4. *(a)* *(i)* Evaluate : 2

$$\lim_{x \to \frac{1}{2}} = \frac{8x^3 - 1}{6x^2 - 5x + 1}$$

(ii) Find $\frac{dy}{dx}$, if

$$y = \sqrt{3x^2 + 2x + 1}$$ 2

(b) If $y = x^3 \log \frac{1}{x}$, then prove that

$$x\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 3x^2 = 0$$ 3

(c) A manufacturer can sell x items per month at a price $p = 300 - 2x$ rupees. Produced items cost the manufacturer y rupees, where $y = 2x + 1000$. How much production will yield maximum profits? 3

UNIT—III

5. *(a)* Verify that the mixed derivatives

$$\frac{\partial^2 z}{\partial x \partial y} \text{ and } \frac{\partial^2 z}{\partial y \partial x}$$

are identical when $z = \frac{x}{x^2 + y^2}$. 3

(b) If

$$f(x, y) = \frac{x + y - 1}{x + y + 1}$$

then find the value of $f_x(2, 1)$. 3

(c) What is total differential? Find the total differential and total derivative, if $u = x^3 y^5$, where $x = t^2$ and $y = t^3$. 4

6. *(a)* Show that the function

$$f(x, y) = \frac{x^2}{xy} + \frac{y^2}{xy}$$

is homogeneous of degree 0. 3

(b) If $z = f(u)$, where u is a function of x and y, then show that

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 0$$

if $u = \frac{x}{y}$. 3

(c) If $f(x, y) = x^2 - 3xy + y^2$, then prove that

$$\frac{\partial f}{\partial x} - \frac{\partial f}{\partial y} = (x - y)\left(\frac{\partial^2 f}{\partial x^2} - \frac{\partial^2 f}{\partial x\,\partial y}\right)$$ 4

UNIT—IV

7. *(a)* Evaluate any *two* of the following : 3½×2=7

(i) $\int \sqrt{x}\,(1 + x)\,dx$

(ii) $\int \frac{x}{\sqrt{x} + 1}\,dx$

(iii) $\int \frac{4x^3}{x^4 + 1}\,dx$

(b) Evaluate any *one* of the following : 3

(i) $\int_0^1 x e^{-x}\,dx$

(ii) $\int_1^2 \frac{dx}{\sqrt{x^2 + x - 2}}$

8. *(a)* Evaluate any *two* of the following : 3½×2=7

(i) $\int_0^1 (\log x)^2\,dx$

(ii) $\int_1^2 x(x + 1)^2\,dx$

(iii) $\int_0^1 \frac{3x^3 - 4x^2 + 1}{\sqrt{x}}\,dx$

(b) Evaluate any *one* of the following : 3

(i) $\int \frac{\log x}{(x + 1)^2}\,dx$

(ii) $\int (2x + 3)\sqrt{x^2 + 3x - 1}\,dx$

UNIT—V

9. *(a)* What is linear programming problem? Write down three applications of linear programming problem. 2+3=5

(b) Solve the following by graphical method : 5

Minimize $Z = 20x + 40y$

subject to the constraints

$$6x + y \geq 18$$
$$x + 4y \geq 12$$
$$2x + y \geq 10$$
$$x, y \geq 0$$

10. *(a)* Write a short note on graphical method of solution of linear programming problem. 4

Or

Define degenerate solution. Find the degenerate solutions of the following system of linear equations :

$$2x_1 + x_2 - x_3 = 2$$
$$3x_1 + 2x_2 + x_3 = 3$$

(b) Solve the following LPP by using simplex method : 6

Minimize $Z = -3x_1 + 4x_2$

subject to the constraints

$$x_1 + x_2 \leq 8$$
$$2x_1 + 5x_2 \leq 22$$
$$x_2 \leq 4$$
$$x_1, x_2 \geq 0$$

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